Ochkov2023 Fig.14

I could not reproduce the solution obtained by roots() shown in the book. The SMath file from the official materials page neither shows a solution.

$S := 12\ \mathrm{m}$ $x_L := 0\ \mathrm{m}$ $y_L := 2\ \mathrm{m}$ $x_R := 10\ \mathrm{m}$ $y_R := 5\ \mathrm{m}$ $M := 2\ \mathrm{kg}$ $m_c := 1\ \frac{\mathrm{kg}}{\mathrm{m}}$ $x_M := 7\ \mathrm{m}$

$y(x; a; x_0; h) := a \cdot \mathrm{ch}\left(\frac{x - x_0}{a}\right) - a + h$ $y'(x; a; x_0) := \mathrm{sh}\left(\frac{x - x_0}{a}\right)$

⊞— Unit-proof integrator, Jacobian and Newton Solver

System of equations

$$Eq := \begin{cases} y_L = y(x_L; a; x_{0L}; h_L) \\ y_R = y(x_R; a; x_{0R}; h_R) \\ y(x_M; a; x_{0L}; h_L) = y(x_M; a; x_{0R}; h_R) \\ S = I\left(\sqrt{1 + \left(y'(x; a; x_{0L})\right)^2}; x; x_L; x_M\right) + I\left(\sqrt{1 + \left(y'(x; a; x_{0R})\right)^2}; x; x_M; x_R\right) \\ -\left(a \cdot m_c\, g_e \cdot y'(x_L; a; x_{0L})\right) + a \cdot m_c\, g_e \cdot y'(x_R; a; x_{0R}) = M\, g_e + m_c\, g_e \cdot S \end{cases}$$

Conversion to vector of residuals (this is also done internally in the NR procedure)

$N := NormRes(Eq)$

Ochkov's Mathcad solution

$$\begin{bmatrix} a \\ x_{0L} \\ h_L \\ x_{0R} \\ h_R \end{bmatrix} := \begin{bmatrix} 6{,}3346578 \\ 3{,}9229196 \\ 0{,}745987 \\ 2{,}2564019 \\ -0{,}3524954 \end{bmatrix} \mathrm{m}$$

Residuals

$$N = \begin{bmatrix} 2{,}64 \cdot 10^{-6}\ \mathrm{m} \\ 1{,}29 \cdot 10^{-5}\ \mathrm{m} \\ 2{,}37 \cdot 10^{-6}\ \mathrm{m} \\ 0{,}00105\ \mathrm{m} \\ -9{,}52 \cdot 10^{-5}\ \frac{\mathrm{m\ kg}}{\mathrm{s}^2} \end{bmatrix}$$

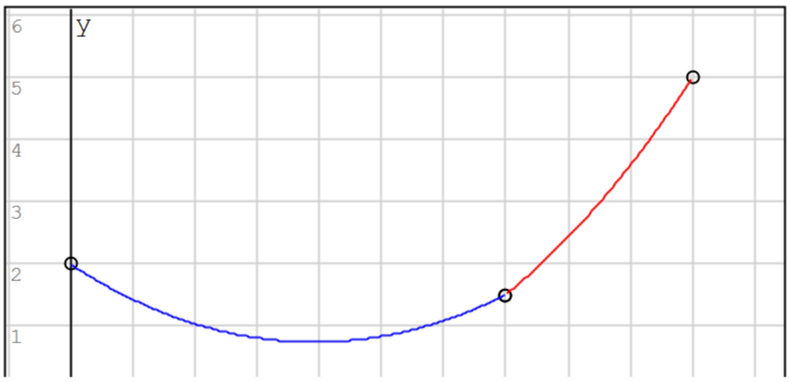
Solution with Kraska's implementation of Newtons method

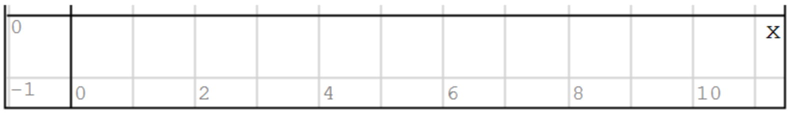
$\mathrm{Clear}(a; x_{0L}; x_{0R}; h_L; h_R) = 1$

$$\begin{bmatrix} a \\ x_{0L} \\ h_L \\ x_{0R} \\ h_R \end{bmatrix} := NR\left(Eq; \begin{bmatrix} a \\ x_{0L} \\ h_L \\ x_{0R} \\ h_R \end{bmatrix}; \begin{bmatrix} 5 \\ 3 \\ 1 \\ 2 \\ -1 \end{bmatrix} \mathrm{m}; 0{,}00001; 0{,}0001\right) = \begin{bmatrix} 6{,}33\ \mathrm{m} \\ 3{,}92\ \mathrm{m} \\ 0{,}745\ \mathrm{m} \\ 2{,}26\ \mathrm{m} \\ -0{,}353\ \mathrm{m} \end{bmatrix}$$

$$N = \begin{bmatrix} -4{,}53 \cdot 10^{-14}\ \mathrm{m} \\ -1{,}97 \cdot 10^{-13}\ \mathrm{m} \\ -3{,}2 \cdot 10^{-14}\ \mathrm{m} \\ -1{,}7 \cdot 10^{-13}\ \mathrm{m} \\ 1{,}89 \cdot 10^{-12}\ \frac{\mathrm{m\ kg}}{\mathrm{s}^2} \end{bmatrix}$$

Fig. 3.15





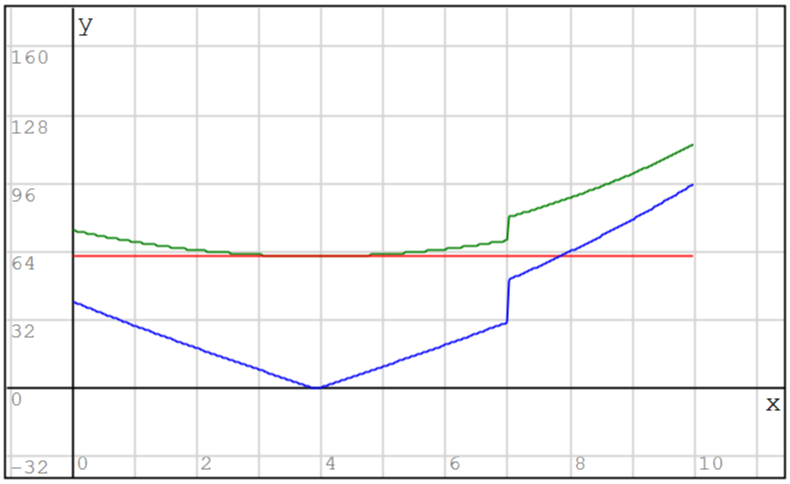
$$\begin{cases} \dfrac{y\left(x\ \mathrm{m};\ a;\ x_{0L};\ h_L\right)}{0 < x\ \mathrm{m} < x_M} \\ \dfrac{y\left(x\ \mathrm{m};\ a;\ x_{0R};\ h_R\right)}{x_M < x\ \mathrm{m} < x_R} \\ \mathrm{augment}\left(\begin{bmatrix} 0 & y_L \\ x_R & y_R \\ x_M & y\left(x_M;\ a;\ x_{0L};\ h_L\right) \\ x_M & y\left(x_M;\ a;\ x_{0R};\ h_R\right) \end{bmatrix};\ \text{"o"}\right) \end{cases}$$

Fig. 3.16

$F_H := a \cdot m_c\ g_e = 62{,}0951\ \mathrm{N}$

$$F_V(x) := F_H \cdot \begin{cases} \text{if } x < x_M \\ \quad y'\left(x;\ a;\ x_{0L}\right) \\ \text{else} \\ \quad y'\left(x;\ a;\ x_{0R}\right) \end{cases}$$

$$F(x) := \sqrt{F_H^{\ 2} + F_V(x)^2}$$



$$\begin{cases} \dfrac{F_V(x\ \mathrm{m})}{x_L < x\ \mathrm{m} < x_R} \\ \dfrac{F_H}{x_L < x\ \mathrm{m} < x_R} \\ \dfrac{F(x\ \mathrm{m})}{x_L < x\ \mathrm{m} < x_R} \end{cases}$$

$y\left(x_L;\ a;\ x_{0L};\ h_L\right)=2\ \mathrm{m}\qquad y\left(x_R;\ a;\ x_{0R};\ h_R\right)=5\ \mathrm{m}$

$y\left(x_M;\ a;\ x_{0L};\ h_L\right)=1{,}5073\ \mathrm{m}\qquad y\left(x_M;\ a;\ x_{0R};\ h_R\right)=1{,}5073\ \mathrm{m}$

$$\int_{x_L}^{x_M}\sqrt{1+y'\left(x;\ a;\ x_{0L}\right)^2}\,\mathrm{d}x+\int_{x_M}^{x_R}\sqrt{1+\left(y'\left(x;\ a;\ x_{0R}\right)\right)^2}\,\mathrm{d}x=12{,}001\ \mathrm{m}$$

$$a\cdot m_c\ g_e\cdot\left|y'\left(x_L;\ a;\ x_{0L}\right)\right|+a\cdot m_c\ g_e\cdot\left|y'\left(x_R;\ a;\ x_{0R}\right)\right|=137{,}2931\ \mathrm{N}$$

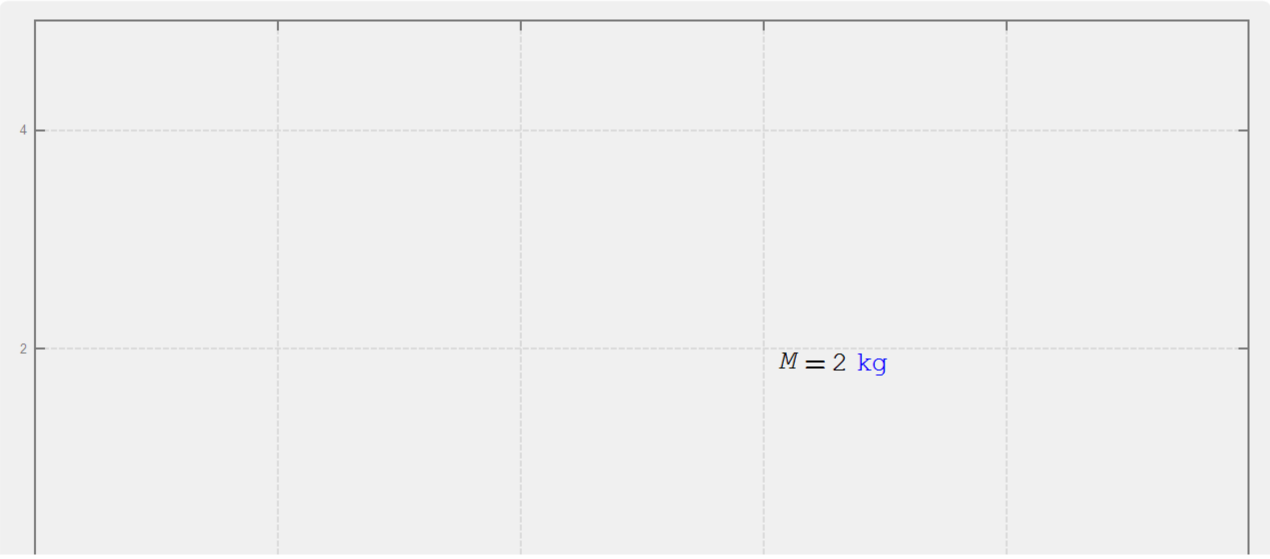
$$M\ g_e+m_c\ g_e\cdot S=137{,}2931\ \mathrm{N}$$

$xx_L:=\left[x_L;\ x_L+\frac{x_M-x_L}{200}\ ..\ x_M\right]\qquad y1:=\mathrm{augment}\left(xx_L;\ \overrightarrow{y\left(xx_L;\ a;\ x_{0L};\ h_L\right)};\ ".";\ 2;\ "red"\right)$

$xx_R:=\left[x_M;\ x_M+\frac{x_R-x_M}{500}\ ..\ x_R\right]\qquad y2:=\mathrm{augment}\left(xx_R;\ \overrightarrow{y\left(xx_R;\ a;\ x_{0R};\ h_R\right)};\ ".";\ 2;\ "blue"\right)$

$xx_L:=\left[x_L;\ x_L+\frac{x_M-x_L}{100}\ ..\ x_M\right]\qquad y3:=\mathrm{augment}\left(xx_L;\ \overrightarrow{y\left(xx_L;\ a;\ x_{0R};\ h_R\right)};\ ".";\ 1;\ "blue"\right)$

$xx_R:=\left[x_M;\ x_M+\frac{x_R-x_M}{50}\ ..\ x_R\right]\qquad y4:=\mathrm{augment}\left(xx_R;\ \overrightarrow{y\left(xx_R;\ a;\ x_{0L};\ h_L\right)};\ ".";\ 1;\ "red"\right)$

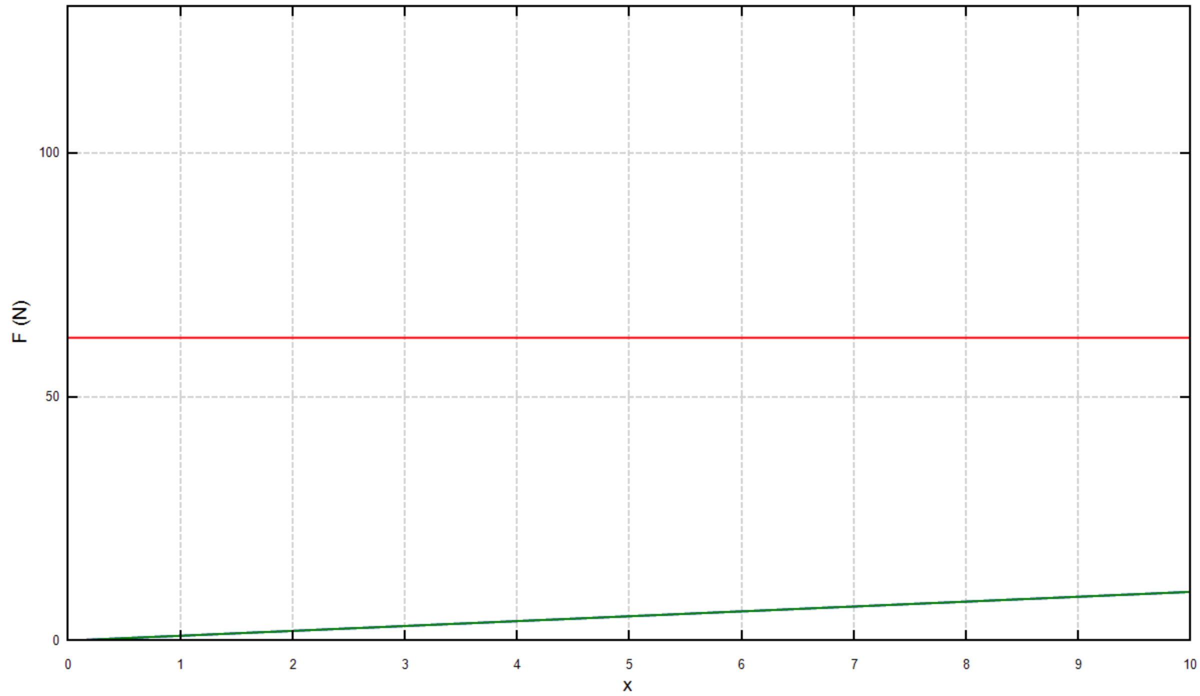


$M=2\ \mathrm{kg}$

$$\begin{cases} y1 \\ y2 \\ \text{augment}\left(x_M;\ y\left(x_M;\ a;\ x_{0L};\ h_L\right) - 0{,}14;\ "."\text{; } 10;\ "black"\right) \\ y3 \\ y4 \end{cases}$$

$F_H := a \cdot m_c\ g_e = 62{,}0951\ N$ $F_V(x) := F_H \cdot \begin{cases} \text{if } x < x_M \\ \quad y'(x;\ a;\ x_{0L}) \\ \text{else} \\ \quad y'(x;\ a;\ x_{0R}) \end{cases}$ $F(x) := \sqrt{{F_H}^2 + F_V(x)^2}$

$X := \left[x_L;\ x_L + 0{,}001\,..\,x_R\right]$ $XF := \text{augment}\left(X;\ \overrightarrow{F(X)}\right)$ $XF_V := \text{augment}\left(X;\ \overrightarrow{F_V(X)}\right)$



$$\begin{cases} XF \\ F_H \\ XF_V \end{cases}$$