IT в инженерных расчетах. Лань. 2023
Рис. 3.2. Поиск размеров идеальной цепи

Перевод на SMath единицы и переработка минимизатора,
M. Kraska, 10/23

Оптимальная цепь

$L := 1\ \text{m} \qquad g_c := 1\ \frac{\text{N}}{\text{m}}$ (1) Исходные данные для расчета

$y(x; a) := a \cdot \text{ch}\left(\frac{x - \frac{L}{2}}{a}\right) - a \qquad y(0\ \text{m}; 1\ \text{m}) = 0{,}1276\ \text{m}$ (2) Цепная линия

$S(a) := \int_{0\ \text{m}}^{L} \sqrt{1 + \left(\frac{d}{dx} y(x; a)\right)^2}\, dx \qquad S(1\ \text{m}) = 1{,}0422\ \text{m}$ (3) Длина кривой

$S(a) := 2 \cdot \sqrt{(y(0\ \text{m}; a) + a)^2 - a^2} \qquad S(1\ \text{m}) = 1{,}0422\ \text{m}$ (4) Длина цепи

$a(S) := \text{solve}(S(a\ \text{m}) = S; a; 0{,}1; 2)\ \text{m} \qquad a(1{,}5\ \text{m}) = 0{,}3082\ \frac{\text{N}}{\text{N}\ \text{m}^{-1}}$ (5)

$F_y(S) := \frac{g_c \cdot S}{2} \qquad F_y(1{,}5\ \text{m}) = 0{,}75\ \text{N}$ (6)

$F_x(S) := a(S) \cdot g_c \qquad F_x(1{,}5\ \text{m}) = 0{,}3082\ \text{N}$ (7)

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F(S) := if IsDefined(S)
           sqrt(Fx(S)^2 + Fy(S)^2)
        else
           F(S)
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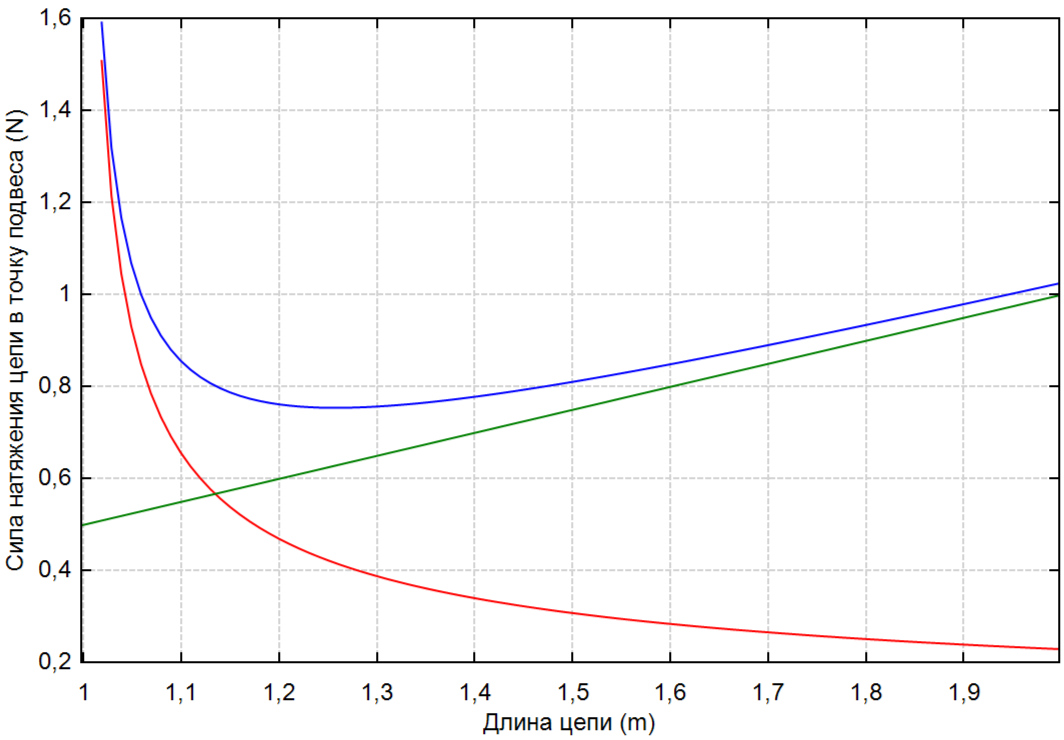
$F(S) := \begin{cases} \sqrt{(F_x(S))^2 + (F_y(S))^2} & \text{if IsDefined}(S) \\ F(S) & \text{otherwise} \end{cases}$ Both versions work

$F(1{,}5\ \text{m}) = 0{,}8109\ \text{N}$ (8)

$F(S) := \sqrt{(F_x(S))^2 + (F_y(S))^2}$ This doesn't work

It is essential that the function has a valid return value if S is not defined.
However, this behaviour should be default.




$\begin{cases} F(x\ \text{m}) \\ F_x(x\ \text{m}) \\ F_y(x\ \text{m}) \end{cases}$

Scalar golden section minimizer

$MinGR(a\#; b\#) :=$

$GR(a\#; b\#) := \frac{3}{2} \cdot b\# - \frac{1}{2} \cdot a\# + \frac{1}{2} \cdot \sqrt{5} \cdot (a\# - b\#)$

$[x_1 := GR(a\#; b\#) \quad y_1 := F(x_1) \quad x_2 := GR(b\#; a\#) \quad y_2 := F(x_2)]$

while $a\# \not\approx b\#$

  if $y_1 < y_2$

    $[b\# := x_2 \quad x_2 := x_1 \quad y_2 := y_1 \quad x_1 := GR(a\#; b\#) \quad y_1 := F(x_1)]$

  else

    $[a\# := x_1 \quad x_1 := x_2 \quad y_1 := y_2 \quad x_2 := GR(b\#; a\#) \quad y_2 := F(x_2)]$

$\frac{a\# + b\#}{2}$

The original function ignored the first argument. F and F(x) are different objects.

$S_{opt} := MinGR(1\ m; 2\ m) = 1,191\ m \qquad F(S_{opt}) = 0,765\ N \qquad F(S_{opt} \cdot 1,01) = 0,761\ N \qquad F(S_{opt} \cdot 0,99) = 0,77\ N$

MinGR(expr, x, a, b, t) Minimizes expr by varying x between a and b by bisection using the golden ratio until they are closer than a tolerance t.

$MinGR(F\#; x\#; a\#; b\#; t\#) :=$

$d := b\# - a\#$

$GR(a\#; b\#) := \frac{3}{2} \cdot b\# - \frac{1}{2} \cdot a\# + \frac{1}{2} \cdot \sqrt{5} \cdot (a\# - b\#)$

$[x_1 := GR(a\#; b\#) \quad x_2 := GR(b\#; a\#)]$

$[y_1 := F\# \big|_{x\# = x_1} \quad y_2 := F\# \big|_{x\# = x_2}]$

Here the assignments were separated because sequence of execution is not guaranteed in a vector

while $(b\# - a\#) > t\# \cdot d$

  if $y_1 < y_2$

    $[b\# := x_2 \quad x_2 := x_1 \quad y_2 := y_1 \quad x_1 := GR(a\#; b\#) \quad y_1 := F\# \big|_{x\# = x_1}]$

  else

    $[a\# := x_1 \quad x_1 := x_2 \quad y_1 := y_2 \quad x_2 := GR(b\#; a\#) \quad y_2 := F\# \big|_{x\# = x_2}]$

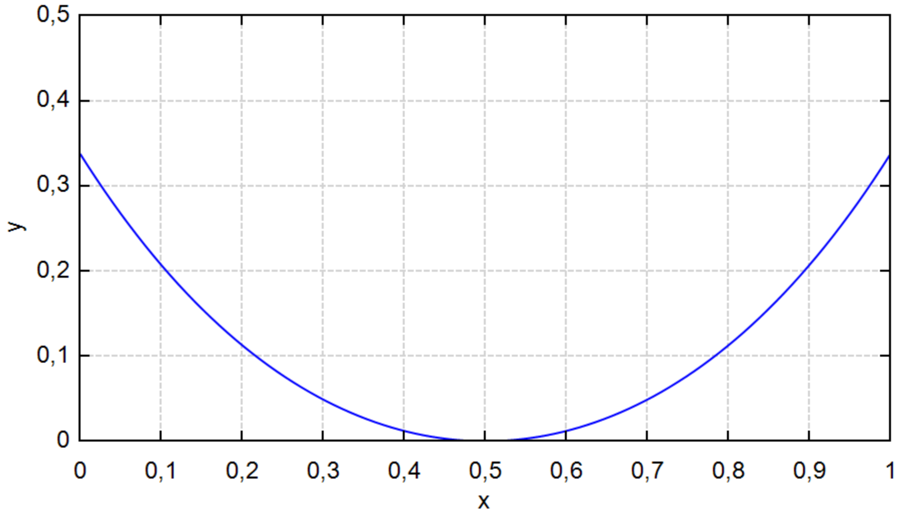
$\frac{a\# + b\#}{2}$

$S_{opt} := MinGR(F(x); x; 1\ m; 3\ m; 0,0001) = 1,258\ m$ $F(S_{opt}) = 0,754\ N$

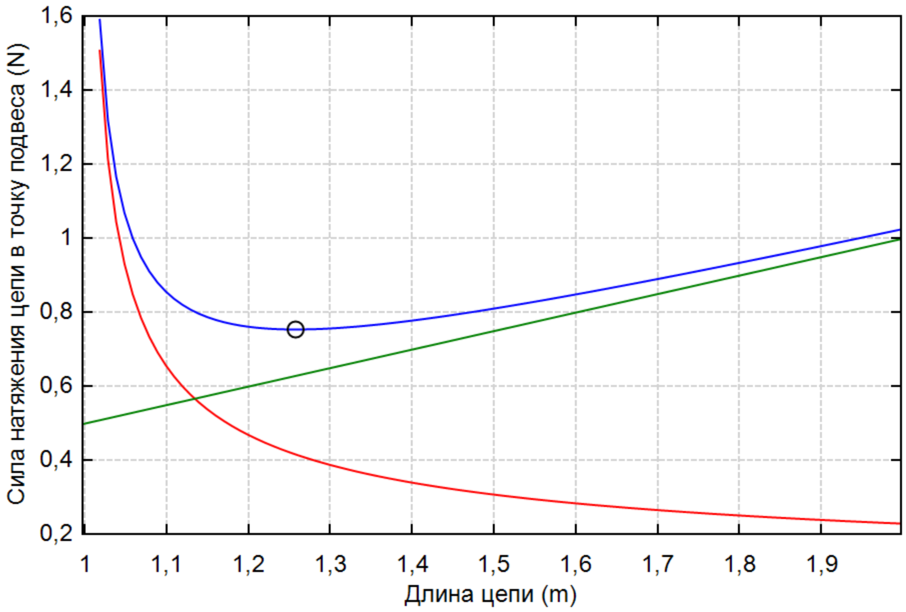
Better result than the original algorithm

$a_{opt} := a(S_{opt}) = 0,4168\ \frac{N}{N\ m^{-1}}$

$tg(\alpha_{opt}) = \frac{F_y(S_{opt})}{F_x(S_{opt})}$ $\alpha_{opt} := arctg\left(\frac{F_y(S_{opt})}{F_x(S_{opt})}\right) = 56,4663\ °$ $\alpha_{opt} = 0,99\ rad$




$y(x\ m; a_{opt})$




$\begin{cases} F(x\ m) \\ F_x(x\ m) \\ F_y(x\ m) \\ [S_{opt}\ F(S_{opt})\ "o"] \end{cases}$