Figures 3.9, 3.10 and 3.11

Complete use of units including in the numerical procedures

Figure 3.9

$h_1 := 10\ \mathrm{m}$ $h_2 := 15\ \mathrm{m}$ $L := 10\ \mathrm{m}$ $S := 20\ \mathrm{m}$ $\mathrm{m} := 1$

$$y\left(x; a; x_0; h_0\right) := a \cdot \mathrm{ch}\left(\frac{x - x_0}{a}\right) - a + h_0$$

$a := 3\ \mathrm{m}$ $x_0 := 4\ \mathrm{m}$ $h_0 := 1\ \mathrm{m}$

$y\left(0; a; x_0; h_0\right) = 4,09\ \mathrm{m}$ $y\left(L; a; x_0; h_0\right) = 9,29\ \mathrm{m}$

$$\int_0^L \sqrt{1 + \frac{d}{d\,x} y\left(x; a; x_0; h_0\right)^2}\, d\,x = 16,2\ \mathrm{m}$$

This representation of the arc length is optional now because the integrator can handle units and can be handled by the solver

$$\sqrt{\left(y\left(0\ \mathrm{m}; a; x_0; h_0\right) - h_0 + a\right)^2 - a^2} = 5,3\ \mathrm{m} \qquad \sqrt{\left(y\left(L; a; x_0; h_0\right) - h_0 + a\right)^2 - a^2} = 10,9\ \mathrm{m}$$

$$\sqrt{\left(y\left(0\ \mathrm{m}; a; x_0; h_0\right) - h_0 + a\right)^2 - a^2} + \sqrt{\left(y\left(L; a; x_0; h_0\right) - h_0 + a\right)^2 - a^2} = 16,2\ \mathrm{m}$$

$\mathrm{Clear}\left(a; x_0; h_0\right) = 1$

$$Eq := \begin{cases} y\left(0; a; x_0; h_0\right) = h_1 \\ y\left(L; a; x_0; h_0\right) = h_2 \\ \sqrt{\left(y\left(0\ \mathrm{m}; a; x_0; h_0\right) - h_0 + a\right)^2 - a^2} + \sqrt{\left(y\left(L; a; x_0; h_0\right) - h_0 + a\right)^2 - a^2} = S \end{cases} \qquad \mathrm{Unknowns}\left(Eq\right) = \begin{bmatrix} a \\ h_0 \\ x_0 \end{bmatrix}$$

⊞— Failed attempts with other solvers

⊞— Unit-proof integrator, Jacobian and Newton Solver

$$\begin{bmatrix} a \\ h_0 \\ x_0 \end{bmatrix} := NR\left(Eq; \begin{bmatrix} a \\ h_0 \\ x_0 \end{bmatrix}; \begin{bmatrix} 2\ \mathrm{m} \\ 1\ \mathrm{m} \\ 4\ \mathrm{m} \end{bmatrix}; 0,001; 0,001\right) = \begin{bmatrix} 2,359\ \mathrm{m} \\ 4,566\ \mathrm{m} \\ 4,398\ \mathrm{m} \end{bmatrix}$$

Figure 3.10

$$\begin{cases} y\left(0; a; x_0; h_0\right) - h_1 \\ y\left(L; a; x_0; h_0\right) - h_2 \\ \sqrt{\left(y\left(0\ \mathrm{m}; a; x_0; h_0\right) - h_0 + a\right)^2 - a^2} + \sqrt{\left(y\left(L; a; x_0; h_0\right) - h_0 + a\right)^2 - a^2} - S \end{cases} = \begin{cases} 4,26 \cdot 10^{-14}\ \mathrm{m} \\ 1,7 \cdot 10^{-13}\ \mathrm{m} \\ 2,1 \cdot 10^{-13}\ \mathrm{m} \end{cases}$$

With the custom integrator, the length can be expressed generically in the equations

$\mathrm{Clear}\left(a; x_0; h_0\right) = 1$

$$\begin{cases} y\left(0; a; x_0; h_0\right) = h_1 \\ y\left(L; a; x_0; h_0\right) = h_2 \end{cases}$$

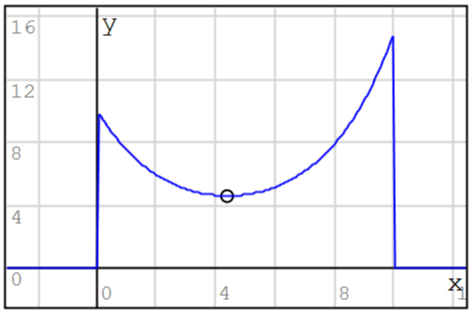
Here, we had to increase the number of intervals in

$$Eq := \left\{ I\left(\sqrt{1+\left(\frac{d}{d\,x}y\left(x;a;x_0;h_0\right)\right)^2};x;0;L;100\right) = S \right.$$

our simple integrator.

$$\begin{bmatrix} a \\ h_0 \\ x_0 \end{bmatrix} := NR\left(Eq; \begin{bmatrix} a \\ h_0 \\ x_0 \end{bmatrix}; \begin{bmatrix} 2\ m \\ 1\ m \\ 4\ m \end{bmatrix}; 0,0001; 0,0001\right) = \begin{bmatrix} 2,358\ m \\ 4,565\ m \\ 4,398\ m \end{bmatrix}$$

Figure 3.11



$$\left\{ \begin{array}{l} y\left(x\ m; a; x_0; h_0\right)\cdot 0<x<\frac{L}{m} \\ \left[\frac{x_0}{m}\ \frac{h_0}{m}\ "o"\right] \end{array} \right.$$