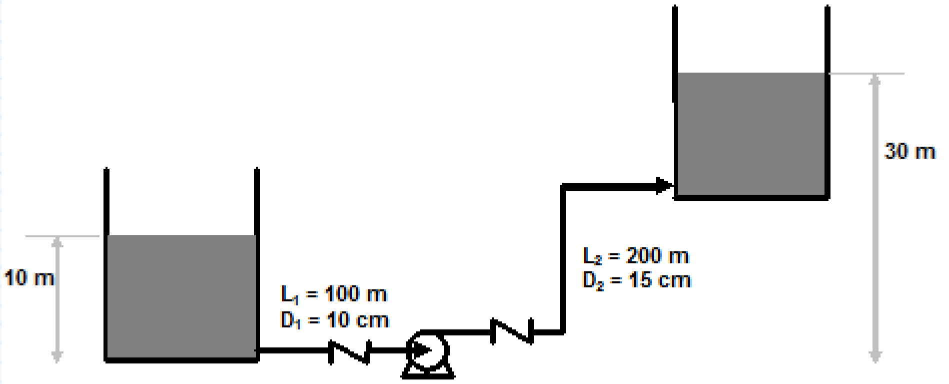
Flowrate Between Two Reservoirs

1. Introduction

A pump transfers from one reservoir to another reservoir according to the schematic below. Check valves are placed either side of the pump and experimental head/flowrate data for the pump is available.



2. Physical Parameters

The following physical paramaters are known:

Pipe Lengths	$L_1 := 50 \cdot m$	$L_2 := 150 \cdot m$
Pipe Diameters	$D_1 := 10 \cdot cm$	$D_2 := 30 \cdot cm$
Pipe Roughness	$e := 0.001 \cdot m$	
Cross Sectional Areas	$A_1 := \frac{\pi \cdot D_1^2}{4}$	$A_2 := \frac{\pi \cdot D_2^2}{4}$
Frictional Loss Coefficients	Check Value	$K_{check} := 3$
	Exit	$K_{exit} := 1$
	Entrance	$K_{ent} := 0.5$
Elevations	$z_1 := 10 \cdot m$	$z_2 := 35 \cdot m$
Liquid Viscosity	$\mu := 10^{-3} \cdot Pa \cdot s$	
Liquid Density	$\rho := 1000 \cdot kg \cdot m^{-3}$	

The following experimental data describe the flowrate-head relationship for the pump.

$$flowrate := \begin{bmatrix} 0 \\ 1000 \\ 2000 \\ 3000 \\ 3500 \\ 4000 \end{bmatrix} \cdot gpm \qquad head := \begin{bmatrix} 230 \\ 228.5 \\ 221 \\ 200.5 \\ 183.5 \\ 157 \end{bmatrix} \cdot ft$$

3. Generating the Pump Curve from Experimental Data

A continuous function that describes the head-flowrate for the pump is formed by fitting a spline to the experimental data.

$$Head_{pump}(Q) := \mathrm{interp}(\mathrm{cspline}(flowrate, head), flowrate, head, Q)$$

4. The Reynolds Number

The function describing the Reynolds number will be a function of both the liquid velocity and pipe diameter.

$$Re(V, D) := \frac{D \cdot V \cdot \rho}{\mu}$$

5. The Friction Factor

We will create a function to calculate the friction factor given the pipe roughness and the instantaneous value of the Reynolds Number. The following structure iteratively solves the Colebrook equation to calculate the friction factor in turbulent flow. PTC Mathcad require a "guess" value of the friction factor to kick-start the numerical algorithms.

Guess value $f_{turb} := 0.1$

Решатель Ограничения

$$\frac{1}{\sqrt{f_{turb}}} = -2 \cdot log\left(\frac{\varepsilon}{3.7} + \frac{2.51}{Re \cdot \sqrt{f_{turb}}}\right)$$

$$f\_turb(Re, \varepsilon) := \mathbf{Find}(f_{turb})$$

The function to calculate the friction factor in laminar flow is far simpler.

$$f\_lam(Re) := \frac{64}{Re}$$

The following function evaluates the turbulent friction factor if Re is above 2300, or the laminar friction factor at all other points

$$friction(Re, \varepsilon) := \begin{array}{l} \text{if } Re > 2300 \\ \quad f\_turb(Re, \varepsilon) \\ \text{else} \\ \quad f\_lam(Re) \end{array}$$

6. The System Curve

The diameter of the piping is not constant. Hence the frictional losses will be written in terms of the volumetric flowrate, Q.

Major Losses:

$$h_L(Q) := friction\left(Re\left(\frac{Q}{A_2}, D_2\right), \frac{e}{D_2}\right) \cdot \frac{L_2}{D_2} \cdot \frac{\left(\frac{Q}{A_2}\right)^2}{2 \cdot g} + friction\left(Re\left(\frac{Q}{A_1}, D_1\right), \frac{e}{D_1}\right) \cdot \frac{L_1}{D_1} \cdot \frac{\left(\frac{Q}{A_1}\right)^2}{2 \cdot g}$$

Minor Losses

$$K_1 := K_{check} + K_{ent} \qquad K_2 := K_{check} + K_{exit}$$

$$K_1 = 3.5 \qquad K_2 = 4$$

$$h_1(Q) := K_1 \cdot \frac{\left(\frac{Q}{A_1}\right)^2}{2 \cdot g} \qquad h_2(Q) := K_2 \cdot \frac{\left(\frac{Q}{A_2}\right)^2}{2 \cdot g}$$

Hence the full system curve is

$$Head_{sys}(Q) := z_2 - z_1 + h_L(Q) + h_1(Q) + h_2(Q)$$

7. Intersection of the System Curve and the Pump Curve

The operating point will be calculated by solving for the flowrate at which the Pump and System curves intersect.

Guess value of the flowrate $Q := 1000 \cdot gpm$

Начальные приближения
Ограничения
Решатель

$$Head_{pump}(Q) = Head_{sys}(Q)$$

$$Q_{op} := \mathbf{Find}(Q)$$

Hence the operating point is $Q_{op} = 0.049 \frac{m^3}{s}$

The intersection of the Pump and System curves can be shown graphically by plotting both curves.

Values of flowrate over which to plot the graph:

$Q := 0 \cdot \mathbf{gpm}, 10 \cdot \mathbf{gpm} .. 4000 \cdot \mathbf{gpm}$