The Transient Draining of Liquid From One Tank to Another with a Pump

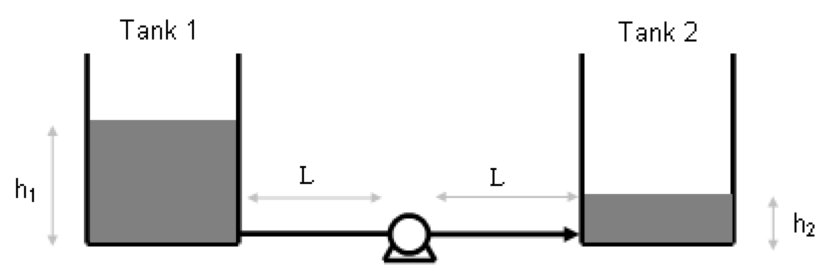
1. Introduction

A pump drains liquid from one tank into another identical tank. It is positioned midway between each tank, connected by a pipe at the bottom of each tank. The flow is opposed by friction in the connecting pipe.

This worksheet models the transient change in liquid height in each tank. It iteratively solves

- The Colebrook equation to describe friction factor in turbulent flow, or the standard eqation that describes the friction factor in laminar flow.
- The Bernoulli equation to find the fluid velocity,
- The Pump Characteristic equation. i.e. a correlation describing head versus flowrate, together with the Continuity equation and the Reynolds Number

at every time step in the solution of the differential equations that describe the transient change in liquid height in both tanks.



2. Physical Parameters

The following physical parameters are known

Liquid Density	$\rho := 1000\ kg \cdot m^{-3}$
Liquid Viscosity	$\mu := 0.0001\ Pa \cdot s$
Pipe Relative Roughness	$e := 0.0001$
Length of pipe	$L := 50\ m$
Diameter of Pipe	$D := 0.05\ m$
Cross-section area of pipe	$A_d := \frac{\pi \cdot D^2}{4}$ $A_d = 0.002\ m^2$

Cross-section area of tank $A_t := 1\ m^2$

Gravitational Constant $g := 9.81\ m \cdot s^{-2}$

Initial Height of Liquid inTank 1 $h1_{init} := 3\ m$

Initial Height of Liquid in Tank 1 $h2_{init} := 0\ m$

3. Head Curve of Pump

The following function is an empirical correlation describing the various of head with flowrate for the pump. Note the dimensions on the contant parameters are to make the function dimensionally consistent.

$$H(Q) := 4 \cdot m - 8 \cdot 10^{-3} \cdot \left(m^{\frac{-5}{2}} \cdot s\right)^2 \cdot Q^2$$

4. Reynolds Number

The liquid velocity V is unknown. The Reynolds Number is hence defined as a function of V.

$$Re(V) := \frac{D \cdot V \cdot \rho}{\mu}$$

5. Continuity Equation

The liquid velocity V is unknown. The Continuity equation is hence defined as a function of V.

$$Q(V) := A_d \cdot V$$

6. Friction Factor

We will create a function to calculate the friction factor given the pipe roughness and the instantaneous of the Reynolds Number.

The following structure iteratively solves the Colebrook equation to calculate the friction factor in turbulent flow.

$f_{turb} := 0.1$

$$\frac{1}{\sqrt{f_{turb}}} = -2 \cdot log\left(\frac{e}{3.7} + \frac{2.51}{Re \cdot \sqrt{f_{turb}}}\right)$$

$f\_turb(Re, e) := \mathbf{Find}\left(f_{turb}\right)$

The function to calculate the friction factor in laminar flow is far simpler.

$$f\_lam(Re) := \frac{64}{Re}$$

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The following function gives the turbulent friction factor if Re is above 2300, or the laminar friction factor at all other points.

$$friction(Re, e) := \begin{array}{l} \text{if } Re > 2300 \\ \quad \| f\_turb(Re, e) \\ \text{else} \\ \quad \| f\_lam(Re) \end{array}$$

7. Calculating Liquid Velocity in the Pipe from the Bernoulli Equation

The instantaneous liquid velocity V is given by an iterative solution of the Bernoulli Equation.

$V := 0.01\ \mathbf{m} \cdot \mathbf{s}^{-1}$

$$h_1 + H(Q(V)) = h_2 + friction(Re(V), e) \cdot \frac{2 \cdot L}{D} \cdot \frac{V^2}{2 \cdot g} + \frac{V^2}{2 \cdot g}$$

$V(h_1, h_2) := \mathbf{Find}(V)$

8. Dynamic Change in Liquid Height in each Tank

The rate of change of liquid height in Tank 1 is $\frac{d}{dt} h_1(t) = -V(h_1(t), h_2(t)) \cdot \frac{A_d}{A_t}$

However the sum of h1(t) and h2(t) is constant $h_1(t) + h_2(t) = h1_{init} + h2_{init}$

Hence $\frac{d}{dt} h_1(t) = -V(h_1(t), h1_{init} + h2_{init} - h_1(t)) \cdot \frac{A_d}{A_t}$

This differential equation will be solved using a simple Euler method.

Time Step $\Delta t := 25\ s$

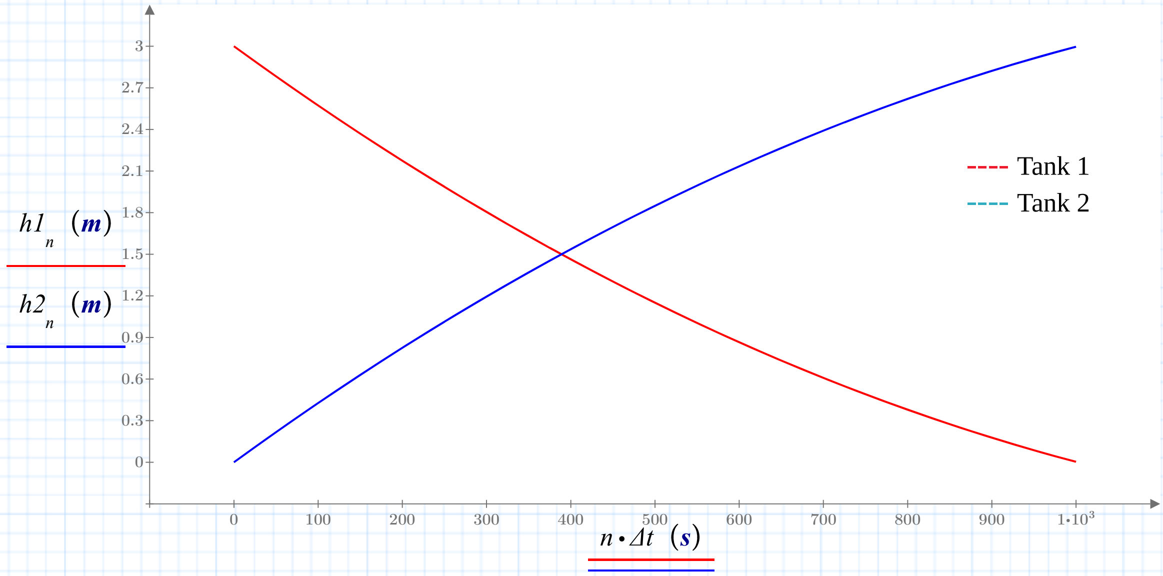
Liquid height in Tank 1 and 2 at t=0 $h1_0 := h1_{init}$ $h2_0 := h2_{init}$

Total number of solution steps $n := 0..40$

Hence the dynamic liquid height in Tank 1 is given by... $h1_{n+1} := h1_n + \Delta t \cdot \left(-V\left(h1_n, h1_{init} + h2_{init} - h1_n\right) \cdot \frac{A_d}{A_t}\right)$

with the dynamic liquid height in Tank 2 given by $h2_n := h1_{init} - h1_n$

Transient Change in Liquid Height



9. Time Required to Drain Tank 1

The total time required to completely drain Tank 1 is hence

$$draining\_time := \frac{A_t}{A_d} \cdot \int_{0\ m}^{h1_{init}} \frac{1}{V\left(h, h1_{init} - h\right)}\, dh$$

$$draining\_time = 1014.8\ s$$