

PHYSICAL QUANTITIES, DIMENSIONS AND UNITS IN MATHEMATICAL PACKAGES

Abstract: *There are described problems and solutions when working with valid dimensions in the medium of mathematical program (engineer calculator) Mathcad.*

Key words: *physical quantities, analysis dimensions, units of measurement, mathematical modeling, Mathcad.*

Анотація: *Описано проблеми та рішення під час роботи з розмірними величинами в середовищі математичної програми (інженерного калькулятора) Mathcad.*

Ключові слова: *фізичні величини, аналіз розмірностей, одиниці вимірювань, математичне моделювання, Mathcad.*

Аннотация: *Описаны проблемы и решения при работе с размерными величинами в среде математической программы (инженерного калькулятора) Mathcad.*

Ключевые слова: *физические величины, анализ размерностей, единицы измерений, математическое моделирование, Mathcad.*

1. Introduction

Programming languages and electronic worksheet which are used for solution of physical tasks operates with simple dimensions (numbers), but not with physical dimensions – mass, rate, energy etc., which, firstly, decreases readability of calculation, and, secondly, causes mistakes.

Mistakes connected with using of invalid measurement units attended all spheres of human relations since olden times. For example, Russian Tsar Peter the Great reproached duke Kurakin in letter (1714, [1, p.9]): “... you’ve sent a list by a hired ship, where proportion is written with Amsterdam pound, but I had always written to you, that there must be English pounds, and I am wondering – why did you disregard it, because now there is a shortage of 10 feet at length and also breadthways”. In our time, time of anthropogenic catastrophes the consequences and detriment can be more considerable.

Thus, New York Times newspaper reports [2] that Boeing airplane which belonged to Canada Airlines, flew from Montreal to Edmonton in 1983, 23 of July. Halfway, being on 12500 meters height airliner had to make an emergency landing in place Gimli because of total fuel consumption. The reason of fuel shortage was a mistake of captain, who calculated fuel mass inaccurately. At that time Canada changed its metric system from British (in this system volume measure is gallon, mass measure is pound) to International SI system (liters and kilograms respectively). For recalculation of required fuel mass it was necessary to multiply fuel gage readings (liters) by respective value of density. The mistake was that captain used invalid value of density (pound/gallon) instead of correct one (kg/l). Consequently the tank of airplane was filled up with only one half of required fuel, which caused the emergency landing.

Such a mistake [3] caused the failure of flight of American satellite Mars Climate Orbiter (MCO). Mars Climate Orbiter was launched in 1998, 11 of December on launch vehicle Delta-2. The apparatus arrived to Mars in 9 months, 23 of September 1999. In a rated time apparatus turned on the engine for braking in 193 km height. In 5 minutes MCO departed to Mars and there were no any signals from it. Data analysis from tracking station revealed that apparatus had flown under the surface on height 57 km only instead of calculated 140 km and was just burned down in the atmosphere (the cost of the apparatus was 125 million \$). Such a big deviation was caused by a mistake of mission specialists: When calculation of

braking impulse one of the groups used British units of measurement (pound - force), and another one used metric ones (newton).

Today every specialist in every field of science and technique can exemplify a lot of mistakes connected with erroneous handling with physical quantities, dimensions or units of measurement. Conversion from manual calculations to computer programming languages had not solved this problem completely.

2. The approaches and methods of work with valid dimensions

Mathematical software packages which are equipped with means of operating with physical quantities (Mathcad, Maple, Derive etc.) accelerate the process of creation of computer calculation programs through automatic recalculation of units of measurement and control of dimensions of variables and functions. This thesis is illustrated on Fig. 1 by the concrete example of solution of the task about the power of the pump transmitting an incompressible liquid in Mathcad.

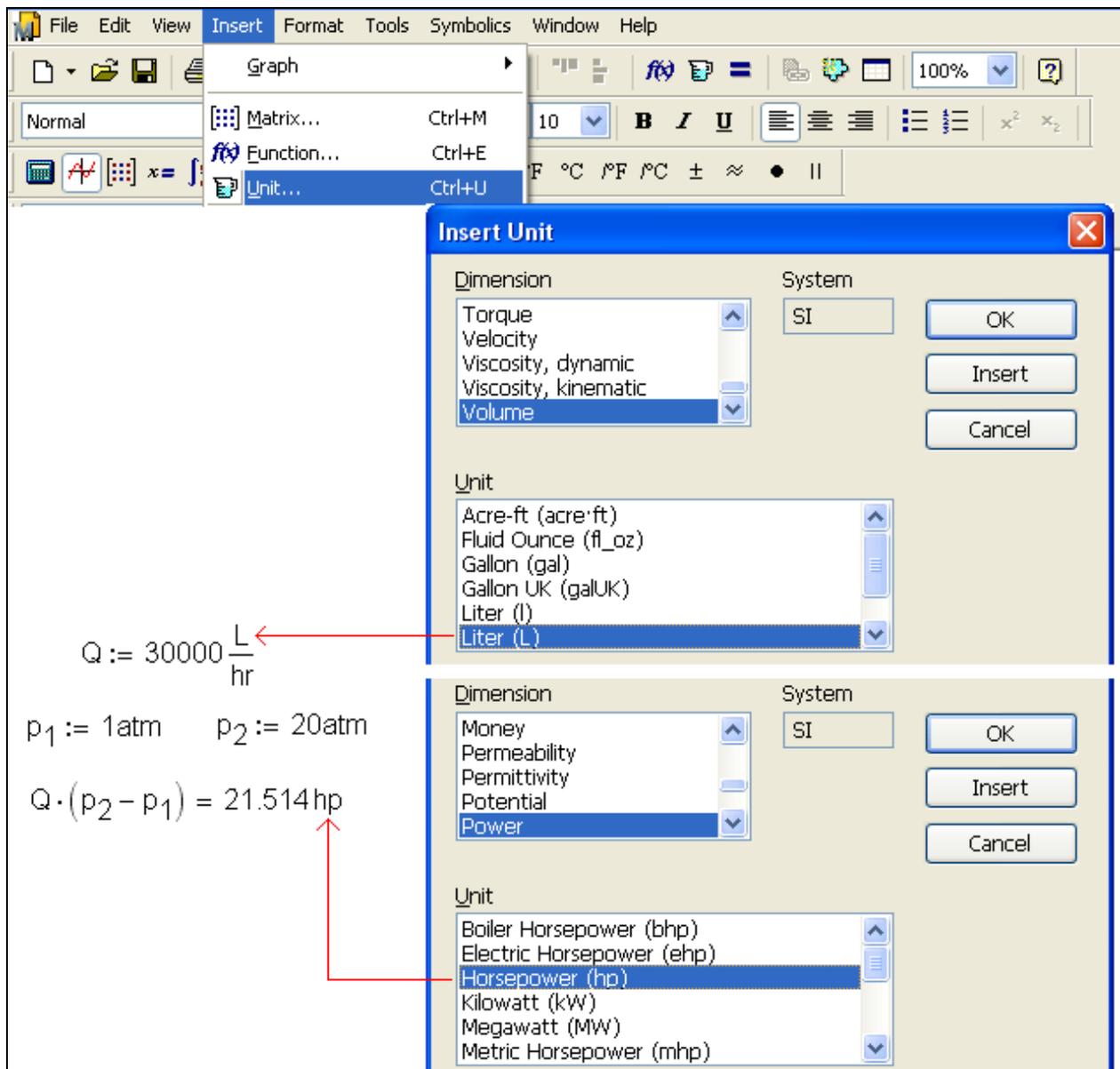
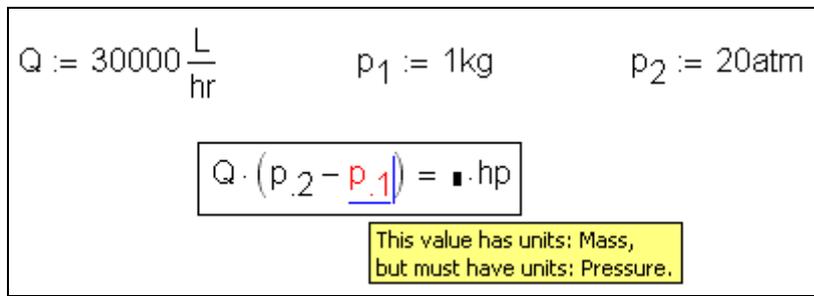


Fig.1. Calculation of pump power in Mathcad

Units of measurement in computer calculations also functions as comments, which indicates the content of one or another variable to user. Together with capabilities of Mathcad to contain superscripts and subscripts, Greek and Cyrillic alphabet and special symbols, this function appreciably improves readability of calculations, approximate its form to handwritten sources of information – handbooks, manual...

Without built-in toolbox of physical quantities in Mathcad the calculation shown on Fig.1 would be necessary supplemented with operators, which recalculate auxiliary but popular units of measurement (L/hr¹, atm, hp) to basic units of one or another system, for example, to International System of Units SI: not liters (L) but cubic meter (m³), not hours (hr) but seconds (s), not standard atmosphere (atm) but pascal (Pa), not horsepower but watt (W) etc.

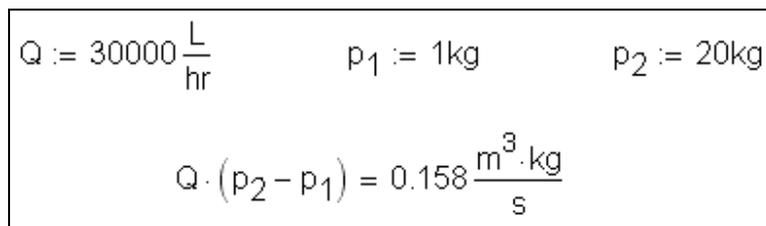
In addition, as it was already noted, there is a control of dimension concordance (not physical quantities but dimensions, and we will talk about it below) during calculations in Mathcad. Thus, Fig. 2 shows the emergency stop of calculation of the pump power in case of mistake in input data – variable p was assigned as mass but not as pressure.



$Q := 30000 \frac{\text{L}}{\text{hr}}$ $p_1 := 1\text{kg}$ $p_2 := 20\text{atm}$
 $Q \cdot (p_2 - p_1) = . \text{hp}$
 This value has units: Mass, but must have units: Pressure.

Fig. 2. Emergency stop of calculation of pump power

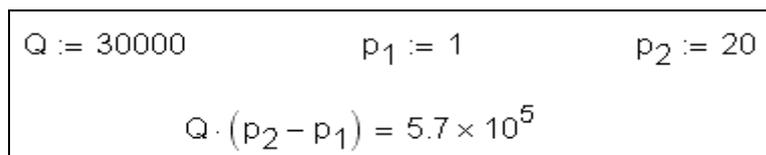
But it is possible to avoid the emergency stop of calculation (fig. 1) not only by correction of the introduced error $p_1 := 1 \text{ kg}$, but also by input of new $p_2 := 20 \text{ kg}$ (fig.3).



$Q := 30000 \frac{\text{L}}{\text{hr}}$ $p_1 := 1\text{kg}$ $p_2 := 20\text{kg}$
 $Q \cdot (p_2 - p_1) = 0.158 \frac{\text{m}^3 \cdot \text{kg}}{\text{s}}$

Fig. 3. Incorrect but troubleproof calculation of the pump power

Emergency stop, when calculation of power, will not occur also when all variables are dimensionless – fig. 4.



$Q := 30000$ $p_1 := 1$ $p_2 := 20$
 $Q \cdot (p_2 - p_1) = 5.7 \times 10^5$

Fig. 4. Dimensionless calculation of pump power

¹ In the task shown on fig. 1 we use international (Latin) units of measurement. But it is possible to operate with users, for example, national units, when connect it with built-in dimensions.

In principle, complete mechanism of dimension control must imply that formulas can contain quantity of concrete dimension – examples shown on fig. 3 and fig. 4 must be finished not just with incorrect answers² (“not-power” on fig. 3 or dimensionless number on fig. 4), but with emergency stop of calculation, as it is shown on fig. 2. Also Mathcad controls the conformity of dimensions and won’t let summarize mass and length, but it does not control the total conformity of physical quantities. And, as it is generally known, different physical quantities can have similar dimensions, which can be the cause of mistakes in calculations in Mathcad and other mathematical packages.

The content of this problem, its partial and probable complete solution is illustrated by more general and understandable task – creation and evocation of the function which recovers volume of cylinder.

Fig. 5 represents this operation in Mathcad.

$$V_C(d, h) := \frac{\pi \cdot d^2}{4} \cdot h$$

$$d := 20\text{cm} \quad h := 15\text{cm}$$

$$V_C(d, h) = 4712.4\text{cm}^3$$

Fig. 5. Conventional creation and “dimensional” evocation of the user’s function

Fig. 5 shows that if variables d and h have dimension of length then function recovers value with dimension “cubed length” (volume) without emergency stop. But the same function failsafely possesses the value of its arguments with units of any other dimensions³ and dimensionless units – fig. 6.

$$V_C(d, h) := \frac{\pi \cdot d^2}{4} \cdot h$$

$$d := 20 \quad h := 15\text{kg}$$

$$V_C(d, h) = 4712.4\text{kg}$$

Fig. 6. Evocation of the cylinder volume function with different dimensions of the argument

And it is not important that function in this case (fig. 6) generates value with dimension which is not equal to cubic length (consequence!). The important object is the reason of such an abnormal situation⁴. For solving this problem let’s remember the remarkable principle which was formulated by Fourier in his classical work “Analytical heat theory”, which was published in 1822 [4]. This principle now is famous as “principle of dimensional uniformity” and it says that any physical equation (which is a sum of two or more terms, as a rule) must contain terms of one dimension only. For illustration of the difference between the algebraic and “physical” equation let’s use an example which was used in Bridgman’s basic

² Everybody, especially programmers, knows that wrong answer is much worse than just a lack of answer.

³ Question concerning possibility of having not only integer-valued but also fractional powers ($m^{0.13}$, $s^{-1.2}$ etc.) in Mathcad is still not solved. Different versions of Mathcad have different reaction on using such units.

⁴ Situation when function’s arguments are dimensionless can be considered (with some stipulation – below in the text) as a normal situation.

work [5]. Analyzing the task about fall of a body under the gravity, Bridgman notices that there are at least two equalities which connect traversed distance (s), velocity (v), time (t) and acceleration of gravity (g):

$$v = gt, s = gt^2 / 2.$$

Now if we will summarize these two equalities, there will appear an equation:

$$v + S = gT + gt^2 / 2,$$

which seems to be verisimilar, but is erroneous – it is impossible to summarize velocity and traversed distance! If we have an available instrument of control of dimensions which is built-in Mathcad we can control the dimension of input value by addition of “dimensional zero”, notably by summation and subtraction of certain value of given dimension. Fig. 7 illustrates the way of creation of user’s function which accepts values with dimensions given by user as an argument.

$V_c(d,h) := \frac{\pi \cdot (d - m + m)^2}{4} \cdot (h - m + m)$
 $d := 20\text{kg} \quad h := 15\text{cm}$
 $V_c(d,h) = \blacksquare\blacksquare$
 This value has units: Mass, but must have units: Length.
 $d := 20$
 $V_c(d,h) = \blacksquare$
 This value has units: Unitless, but must have units: Length.
 $d := 20\text{cm}$
 $V_c(d,h) = 4712.4\text{cm}^3$

Fig. 7. Creation and activation of user's dimensional function

Fig. 7 shows that anew created function which recovers cinder volume possesses the value of arguments of length dimension only. And this function recovers values of cubed length dimension (volume) only.

The following example shows that Mathcad controls the compliance of dimension of physical quantities but not the physical quantities proper.

Fig. 8 shows that when invoking function which recovers the cylinder volume there can appear an error if we will swap the function’s arguments – put the height of cylinder in place of its base and put the base in place of height. This error can be avoided by strengthening of control: to aim it not to dimensions but to physical quantities.

$$V_C(d, h) := \frac{\pi \cdot (d - m + m)^2}{4} \cdot (h - m + m)$$

d := 20cm h := 15cm

$$V_C(h, d) = 3534.3 \text{ cm}^3$$

Fig. 8. Error when evocation of the user's function

The enquiry is that the units of measurement in Mathcad should be concatenated not to dimensions (length – meters, feet, miles etc., mass – kilograms, pounds etc.) but to physical quantities. For the first time in the most complete execution this idea was effectively evolved in work by G. Hantley [6], who particularly offered to use “vectorial dimensions” that means to assign different dimension symbols to the multidirectional quantity.

At that it is necessary to accept that meters of our cylinder base and meters of its height are the different quantities, and this difference must be evidenced, for example, by impossibility of its summation: there will appear an error as in case of summation or subtraction of pressure and mass (fig. 2).

There are eight dimensions in Mathad – length, time, mass, current strength, temperature, luminous intensity, amount of matter (“the magnificent seven” of SI) and cost (in 14th Mathcad version, the last one by the time of writing this paper). If it is necessary to input another dimension, for example, amount of information with dimensions of bit, byte etc., or cost (in the previous 13 versions of Mathcad) we have to reserve one of the free dimensions. Fig. 9 illustrates how to divide two identical dimensions into two different physical quantities and to assign to it two different units of length.

$$m-d := m \quad m-h := cd$$

$$V_C(d, h) := \frac{\pi \cdot \left(\frac{d + m-d}{m-d} - 1 \right)^2}{4} \cdot \left(\frac{h + m-h}{m-h} - 1 \right) \cdot m^3$$

d := 0.2m-d h := 0.15m-h

$$V_C(d, h) = 4712.4 \text{ cm}^3$$

$V_C(h, d) = \blacksquare$

This value has units: Luminous Intensity, but must have units: Length.

Fig. 9. Calculation of cylinder volume with diameters of base (the first physical quantity) and height (the second physical quantity)

Now (fig. 9) intentional or unintentional swapping of arguments of function, which recovers the cylinder volume, will be stopped with an error report, which can be deciphered in this way: “Wrong physical quantity – cylinder height instead of its base!”

Actually error report says (fig. 9) that “There must be length but not luminous intensity⁵!” But if we will remember that “free” candela in this calculation was assigned as a wanting physical quantity, everything is logical.

All the foregoing reasonings can be taken as a certain “exotica⁶” in field of theory of dimensional (physical) quantities and as an attempt of application of this theory to the computer technologies. But here is a real thermodynamic task, and when its solution in Mathcad it will be necessary (or at least it will be advisable) to operate with different physical quantities which have the same dimension (in this case it is mass).

We need to calculate the coefficient of thermal efficiency of a steam-to-gas heat-and-power engineering cycle. For this purpose it is necessary to operate with functions which recover thermodynamic behaviors of a working substance. These functions can be provided with help of WaterSteamPro program package – fig. 10.

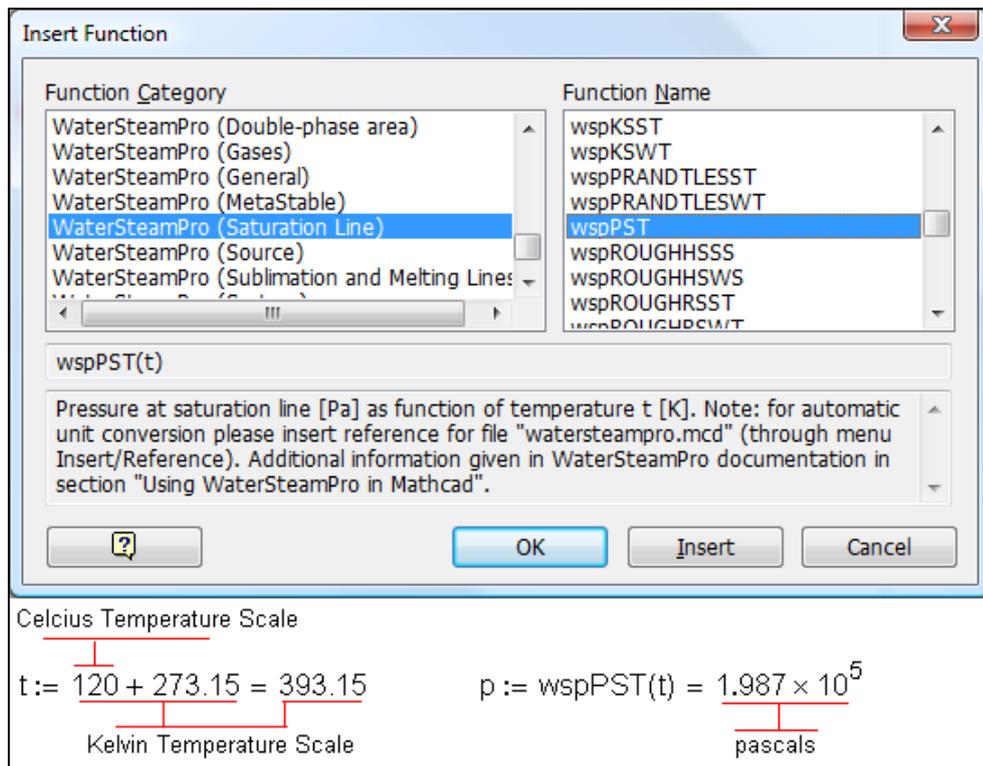


Fig. 10. Functions of WaterSteamPro package in Mathcad environment

Fig. 10 illustrates the way of calculation of water boiling temperature at given pressure by call of one of functions of WaterSteamPro package – wspTSP function. Meanwhile all quantities still are dimensionless, it is purported that pressure is expressed in pascal, temperature is gauged by Kelvin scale (expressed in kelvin). All functions of WaterSteamPro package operate with dimensionless arguments and

⁵ Unit of luminous intensity candela cd, as a rule, is used for imitation of unit of an absent measurement.

⁶ If we accept diameter and height of cylinder as different physical quantities and “report” it to the computer to secure against errors, we should admit that cylinder volume must be measured not in m³, but in m².m, and these two meters are related to the different quantities.

call dimensionless quantities⁷, which conforms to basic units of measurement in SI: meters, kilograms, joules (but not kilojoules), pascals (but not megapascals) etc.

But, as it was said in commencement of this text, mathematical program Mathcad is equipped by instruments for operation with dimensional units, and this mechanism must be used in solution of our task about steam-to-gas cycle and generally in heat-and-power engineering calculations.

For performing the functions of WaterSteamPro package as a dimensional functions it is necessary to make a link in concrete Mathcad document (Reference) to the watersteampro.mcd file which is built-in WaterSteamPro package – fig. 11.

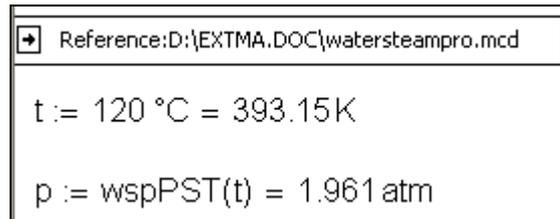


Fig. 11. “Dimensional” functions of WaterSteamPro package in Mathcad

What does file watersteampro.mcd contain?!

First, it contains operators which implement the national dimensions into the calculation and different constants which are required for heat-and-power engineering calculations (universal gas constant, temperature and pressure of triple and critical points of water and water steam etc.). Second, and main, watersteampro.mcd file contains operators which turn dimensionless functions of WaterSteamPro package into dimensional in Mathcad environment.

But there is another way – not to create link to file warwesteampro.mcd which redefines all functions of the package, but to insert operators which redefine only some of the functions which are necessary for the concrete thermotechnical calculation, taking in consideration the thesis which says that certain different physical units have the same dimension. Particularly, in our task about thermodynamic efficiency of steam-to-gas cycle there will be two different physical units – mass of gas and mass of water / water steam, which have the same dimension of mass.

For when calculation we will need the following functions which are “dimensionally” and “physically⁸” are redefined in the Mathcad-document – fig. 12.

For gas (in our calculation – for air):

- wspgHGST: enthalpy (H) of gas depending on its specification (GS)⁹ and temperature (T);
- wspgSGSPT: entropy (S) of a gas depending on its specification(GS) and pressure (P);
- wppgTGSPT: temperature (T) of a gas depending on its specification (GS), pressure (P) and entropy (S).

⁷These functions are programmed on “dimensionless” programming language C.

⁸ “Physically” it means that kilogram of gas (kkg) and kilogram of water/steam (kgws) are related to the different dimensions: kkg=cd (also fig. 9) and kgws=kg which, as it is shown below, intensifies control of correct calculations.

⁹ Specification of the gas depends on its chemical composition. In our case it is air (GS:=“air”), but it can be any other gas – natural gas, domestic gas etc.

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kgg := cd      J/kgg :=  $\frac{J}{\text{kgg}}$       J/(kgg K) :=  $\frac{J}{\text{kgg K}}$ 

wspgHGST(gas_specification, t) := wspgHGST[user](gas_specification,  $\frac{t + K}{K} - 1$ ) · J/kgg

wspgSGSPT(gas_specification, p, t) := wspgSGSPT[user](gas_specification,  $\frac{p + Pa}{Pa} - 1, \frac{t + K}{K} - 1$ ) · J/(kgg K)

wspgTGSPS(gas_specification, p, s) := wspgTGSPS[user](gas_specification,  $\frac{p + Pa}{Pa} - 1, \frac{s + J/(\text{kgg K})}{J/(\text{kgg K})} - 1$ ) · K

kgws := kg     J/kgws :=  $\frac{J}{\text{kgws}}$      J/(kgws K) :=  $\frac{J}{\text{kgws K}}$ 

wspHPT(p, t) := wspHPT[user]( $\frac{p + Pa}{Pa} - 1, \frac{t + K}{K} - 1$ ) · J/kgws

wspSPT(p, t) := wspSPT[user]( $\frac{p + Pa}{Pa} - 1, \frac{t + K}{K} - 1$ ) · J/(kgws K)

wspPST(t) := wspPST[user]( $\frac{t + K}{K} - 1$ ) Pa

wspXSTS(t, s) := wspXSTS[user]( $\frac{t + K}{K} - 1, \frac{s + J/(\text{kgws K})}{J/(\text{kgws K})} - 1$ )

wspHSTX(t, x) := wspHSTX[user]( $\frac{t + K}{K} - 1, x$ ) · J/kgws

wspHSWT(t) := wspHSWT[user]( $\frac{t + K}{K} - 1$ ) · J/kgws

wspHSST(t) := wspHSST[user]( $\frac{t + K}{K} - 1$ ) · J/kgws

wspSSWT(t) := wspSSWT[user]( $\frac{t + K}{K} - 1$ ) · J/(kgws K)

wspHPS(p, s) := wspHPS[user]( $\frac{p + Pa}{Pa} - 1, \frac{s + J/(\text{kgws K})}{J/(\text{kgws K})} - 1$ ) · J/kgws

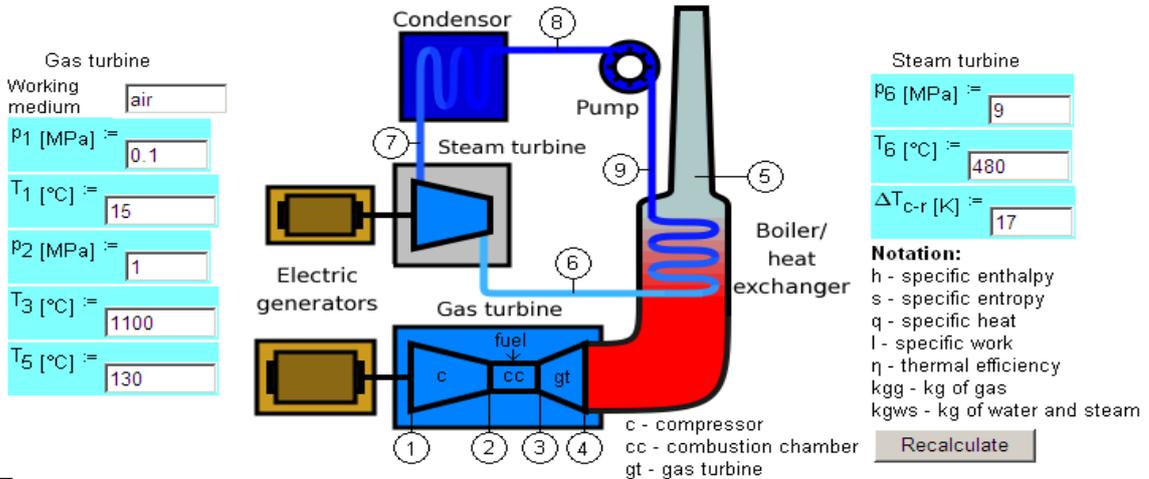
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Fig. 12. Overloading of certain functions of WaterSteamPro package

For water and water steam:

- wspHPT: enthalpy (H) of water and steam depending on pressure (P) and temperature (T);
- wspSPT: entropy (S) of water and steam depending on pressure (P) and temperature (T);
- wspPST: pressure (P) of water on the water saturation line (here S – saturation) depending on temperature (T);
- wspXSTS: dryness factor (X) of wet steam (S – см. выше) depending on temperature (T) and entropy (S);
- wspHSTX: enthalpy (H) of wet steam (S) depending on temperature (T) and dryness factor (X);
- wspHSWT: enthalpy (H) of water (W) on the water saturation line (S) depending on temperature (T);
- wspHSST: enthalpy (H) of steam (S) on the saturation line (S) depending on temperature (T);
- wspSSWT: entropy (S) of water (W) on the saturation line (S) depending on temperature.

Thermal efficiency of **Combined cycle**: Rankin cycle (steam turbine) + Brayton cycle (gas turbine)



Ideal Gas turbine calculation

af = "air" $h_1 := \text{wspgHGST}(af, T_1) = 288.56 \text{ kJ/kgg}$ $s_1 := \text{wspgSGSPT}(af, p_1, T_1) = 6.83 \text{ kJ/(kgg K)}$

$s_2 := s_1$ $T_2 := \text{wspgTGSPS}(af, p_2, s_2) = 279.02 \text{ °C}$ $h_2 := \text{wspgHGST}(af, T_2) = 557.39 \text{ kJ/kgg}$

$p_3 := p_2$ $s_3 := \text{wspgSGSPT}(af, p_3, T_3) = 7.847 \text{ kJ/(kgg K)}$ $h_3 := \text{wspgHGST}(af, T_3) = 1483.61 \text{ kJ/kgg}$

$p_4 := p_1$ $s_4 := s_3$ $T_4 := \text{wspgTGSPS}(af, p_4, s_4) = 498.08 \text{ °C}$ $h_4 := \text{if}(T_4 > T_6, \text{wspgHGST}(af, T_4), \text{"Error"}) = 790.87 \text{ kJ/kgg}$

$q_{cc} := h_3 - h_2 = 926.22 \text{ kJ/kgg}$ $l_{\text{gas turb}} := h_3 - h_4 = 692.74 \text{ kJ/kgg}$ $l_{\text{compressor}} := h_2 - h_1 = 268.83 \text{ kJ/kgg}$

$$\eta_{\text{gas turb}} := \frac{l_{\text{gas turb}} - l_{\text{compressor}}}{q_{cc}} = 45.77\%$$

Mass of gas (air)

Ideal Steam turbine calculation

$h_6 := \text{wspHPT}(p_6, T_6) = 3336.33 \text{ kJ/kgws}$ $s_6 := \text{wspSPT}(p_6, T_6) = 6.593 \text{ kJ/(kgws K)}$

$T_7 := T_1 + \Delta T_{c-p} = 32 \text{ °C}$ $p_7 := \text{wspPST}(T_7) = 4.76 \text{ kPa}$ $s_7 := s_6$ $x_7 := \text{wspXSTS}(T_7, s_7) = 77.12\%$

$h_7 := \text{wspHSTX}(T_7, x_7) = 2004.37 \text{ kJ/kgws}$ $T_8 := T_7$ $h_8 := \text{wspHSWT}(T_8) = 134.11 \text{ kJ/kgws}$

$p_9 := p_6$ $s_8 := \text{wspSSWT}(T_8) = 0.464 \text{ kJ/(kgws K)}$ $s_9 := s_8$ $h_9 := \text{wspHPS}(p_9, s_9) = 143.13 \text{ kJ/kgws}$

$q_{\text{boiler}} := h_6 - h_9 = 3193.2 \text{ kJ/kgws}$ $l_{\text{steam turb}} := h_6 - h_7 = 1331.96 \text{ kJ/kgws}$ $l_{\text{pump}} := h_9 - h_8 = 9.023 \text{ kJ/kgws}$

$$\eta_{\text{steam turb}} := \frac{l_{\text{steam turb}} - l_{\text{pump}}}{q_{\text{boiler}}} = 41.43\%$$

Mass of Water and Steam

Ideal Combined cycle calculation

$h_5 := \text{wspgHGST}(af, T_5) = 404.51 \text{ kJ/kgg}$

Heat balance of the boiler $m \cdot (h_4 - h_5) = h_6 - h_9$ $m := \frac{h_6 - h_9}{h_4 - h_5} = 8.26 \text{ kgg/kgws}$

$q_{cc} := m \cdot (h_3 - h_2) = 7655.13 \text{ kJ/kgws}$ $l_{\text{gas turb}} := (h_3 - h_4) - (h_2 - h_1) = 423.91 \text{ kJ/kgg}$

$$l_{\text{steam turb}} := (h_6 - h_7) - (h_9 - h_8) = 1322.94 \text{ kJ/kgws}$$

$$\frac{m \cdot l_{\text{gas turb}}}{l_{\text{steam turb}}} = 2.648$$

$$\eta_{\text{combined cycle}} := \frac{m \cdot l_{\text{gas turb}} + l_{\text{steam turb}}}{q_{cc}} = 63.05\%$$

$$h_1 - h_6 = \blacksquare$$

$$l_{\text{gas turb}} + l_{\text{steam turb}} = \blacksquare$$

The units in this expression do not match.

The units in this expression do not match.

Useful Added Error Messages!

Insert Unit

Dimension

Mass:

- Mass1
- Mass2
- Mass3
- Mass4

Unit

- Gram (gm)
- Kilogram (kg)
- Metric ton (tonne)
- Milligram (mg)
- Ounce (oz)
- Pound (lb)

Fig. 13. Calculation of steam-to-gas cycle

Fig. 12 shows that twelve functions of WaterSteamPro package (and there are more than 300 of them in common) which are necessary for the coefficient of thermal efficiency of the steam-to-gas cycle are not redefined but anew created. In the calculation there will be pairs of functions: $\text{wspXXX}_{[\text{USER}]}$ – initial dimensionless functions and wspXXX (without any subscript) – anew created functions on base of previous “dimensional” functions and in these latter functions will have control of argument’s dimensions (in the place of temperature there can be only temperature, in the place of pressure – only pressure etc.)

ant will output dimensional quantities – enthalpy (kJ/kg^{10}), entropy ($\text{kJ}/(\text{kgK})$), and there will be not just “divide by kilogram” but “divide by kilogram of gas” or “divide by kilogram of water / water steam”.

Now there are all necessary functions with *wspg* and *wsp* subscripts for our thermotechnical calculation in Mathcad environmental and it is easy to realize the calculation of our steam-to-gas cycle – fig. 13.

In the calculation which is shown on fig. 13 there is not only concretization of kilogram (*kgg* and *kgws*) but control of correct work with physical quantities (enthalpy, entropy). Though at the end of calculation there are two emergency stops (difference of enthalpy of work cycle in point 1 and point 6 and sum of specific work of gas and steam turbines).

In the absence thereof physical quantities with mass dimension (fig. 12) there would not be any emergency stops in these operators, which could cause errors in calculations with different consequences (examples are described at the beginning of the text).

Real combined-cycle plant (but not ideal as in our task) there are more than two working substances. Alongside with steam and air it is necessary to take into account thermodynamic properties of fuel which is directed to combustion chamber of gas turbine (natural gas of different chemical composition, combustion products etc.). Therefore it is better to use more than two (*kgg* and *kdws*) units of mass and the other units of measurement. Dialog box in the right lower corner (fig. 13) is our wish to creators of the following versions of engineering calculators.

One of the solving of problem of different physical quantities with identical dimensions in Mathcad can be provided by complementary inlay Quality in dialog window Properties which appears when press right mouse bottom on the name of anew created function. In this inlay user can write or – even better – paste the required dimension (basic – mass,..., amount of matter¹¹ or composite one force, power etc.) from Dimension window. And it is possible to supplement the name of unit with number 1, 2, 3 etc., which differs one physical quantity from another with the same dimension. This technology is illustrated on fig. 14.

In the calculation (fig. 13) dimensionless variable *m* (quotient of mass flow of gas in the gas turbine to mass flow of steam in the steam turbine) turned into dimensional one. And, as we showed, it is good, because the less dimensionless units in the calculation the wider will be the scope for instruments of dimensional control. But there are the other dimensionless units - dryness factor of steam (quotient of mass of dry vapor to mass of moisture (drops) in it) in unit of volume and three values of coefficient of thermal efficiency (η_t – quotient of effective work to the spent heat). If we will describe numerator and denominator in these fractions (quotient) as identical physical quantities, these fractions will be dimensionless too. If we will describe numerator and denominator in these fractions as different physical quantities (kg of water and kg of steam, joule of work and joule of heat¹²), these quantities (*x* and η_t) will have not just dimension but different dimensions and it will be impossible to sum it (for example)

¹⁰ It is more easy and short to write J/g (divide joule by gram), but we are accustomed to “kilo” in numerator and denominator and it is difficult to strike it off.

¹¹ This “magnificent seven” of SI in 14 version of Mathcad is turned into “eight” – package has units of cost π , and users can relate it to their national currency (€, \$) or... use it for another purpose as we used candela (cd) for designation of “another” kilograms.

¹² This differentiation of work and heat returns us to the base *c* thermodynamic conception which intertwines these two physical quantities to the equal dimensional unit: $\text{kgs}\cdot\text{m}$ (work), kal (heat), J (work an heat).

intentionally or accidentally in Mathcad or any other mathematical package which provides the mechanism of dimensional quantities not partially (as it is now) but completely (as it is shown on fig. 14).

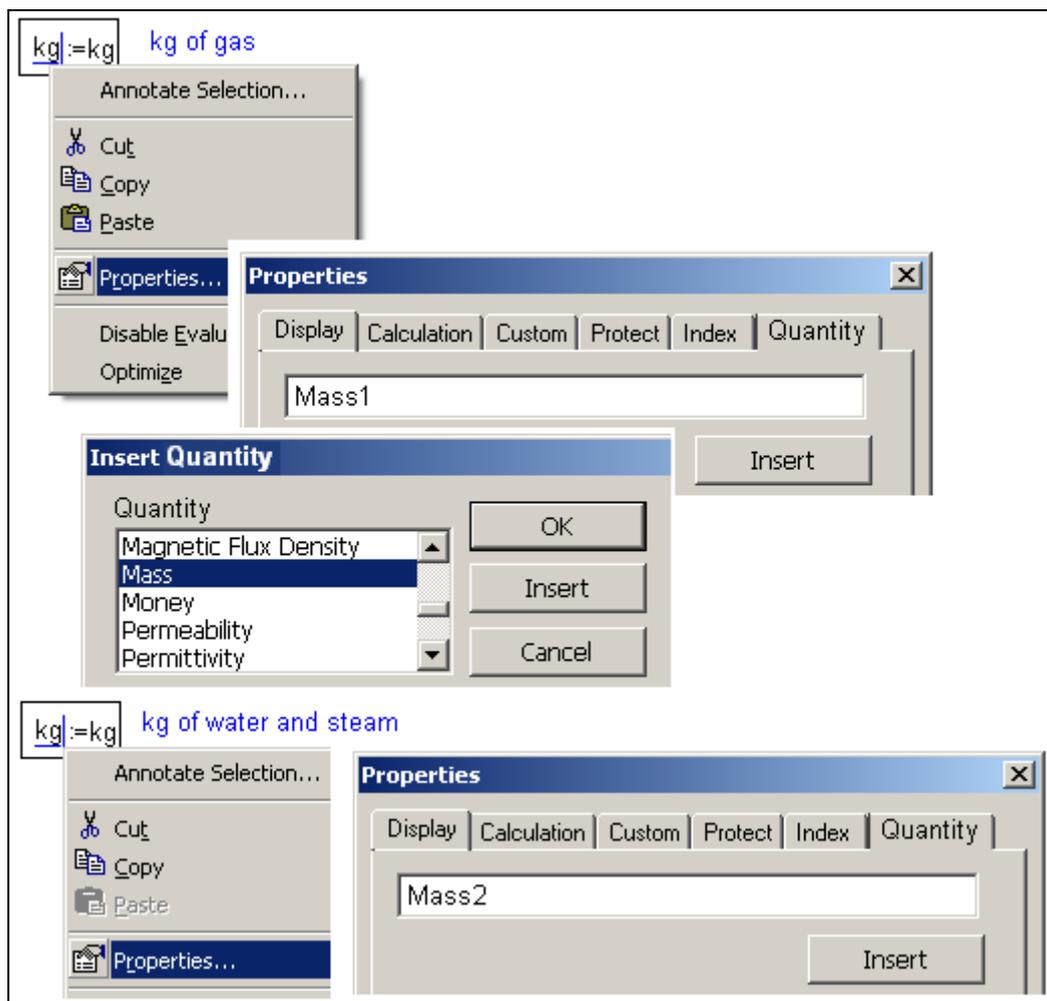


Fig.14. Proposal for development of instruments “Physical quantity – Dimension – Unit of measurement” in Mathcad

3. Conclusions

In conclusion we will remember about such a base physical quantity as plane angle or solid angle. This quantity, as a rule, starts the list of dimensions which are described in SI: first – angles, second - the magnificent seven of SI, after this – units which are comprised by the base units for force, power etc. Though it is declared that angles are dimensionless unit it is remarked that its dimension is m/m (plane angle, radian). We will not go into the theory, but let’s ask ourselves if it is possible to sum radian and steradian?! In the old versions of Mathcad such summation used to be stopped with an error message because radian was a dimensionless unit ($\text{rad} = 1$) and steradian was dimensional ($\text{sr} = \text{sr}$). Later programmers of the package understood that it is illogical and in 13 and 14 versions of Mathcad they have made... steradian dimensionless too ($\text{sr} = 1$), decided, evidently, that m^2/m^2 can be canceled as it was

done¹³ for m/m for plant angle. But imagine what if plant angle would be dimensional (rad = rad)?! In this case sinus, cosines and the other trigonometric functions would depend on dimensional values... And sinus would have dimension m/m or ratio of new meters (cathetus length) to new meters (hypotenuse length)... This absolute insanity should be stopped! And it was done by built-in only partial control of dimensions in the mathematic packages, purporting that this control has no concrete limits.

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Стаття надійшла до редакції 07.10.2008

¹³ Question about shortening in compound units of measurement is rather tangled. Thou, in Mathcad, when type-out, specific work and specific energy have dimension m^2/s^2 which has to be changed on more readable kJ/kg, returning mass into dimension. Such shortage bereaves quantities of its physical meaning and this mistake is made not only by “silly” computer but by “clever” user. Here is one of the examples. In all manuals thermal conductivity material is presented with $W/(m \cdot K)$ dimension (length is canceled), but right form is $W \cdot m/(m^2 \cdot K)$. In this case there are different meters (thickness and surface) and it should not be canceled.